Data Structures
CSCI C343, Fall 2015

Quiz 1

Name: _____

This quiz has 3 questions, for a total of 30 points.

1. [9 points] Suppose that `L` is a Python `list` (array) of length n. Categorize the worst-case execution time of the below expressions as either

   1. $O(1)$
   2. $O(\lg n)$
   3. $O(n)$
   4. $O(n^2)$

   Label each operation with the above item number.

   - `L.insert(1, 42)`
   - `L[n-1]`
   - `1 in L`

   Solution:

   - (3), `L.insert(1, 42)` is $O(n)$, (3 points)
   - (1), `L[n-1]` is $O(1)$, (3 points)
   - (3), `1 in L` is $O(n)$, (3 points)

2. [11 points] Let $f(n) = 5n + 10$ and $g(n) = n^2$. Give the definition of Big-O and prove that $f(n) \in O(g(n))$.

   Solution: Definition of $O(g(n))$: (3 points)

   $$O(g(n)) = \{f(n) \mid \exists n_0.\forall n \geq n_0.\exists c.\ 0 \leq f(n) \leq c\, g(n)\}$$

   We need to choose a c such that cn^2 becomes greater than $5n + 10$ at some point. We choose $c = 1$ (3 points, there other valid choices) because n^2 is going to dominate $5n$ regardless of the choice of c, and $c = 1$ is the easiest choice. Next we need to find out at what point n^2 is equal to or bigger than $5n + 10$, so we chart those out:

n	$5n + 10$	n^2
4	30	16
5	35	25
6	40	36
7	45	49

So it looks like $n_0 = 7$ is a good choice (3 points, there other valid choices). We are now ready to give the proof.

To show that $5n + 10 \in O(n^2)$, we need to show that

$$\exists n_0.\forall n \geq n_0.\exists c.\ 0 \leq 5n + 10 \leq c\,n^2$$

We choose $n_0 = 7$ and $c = 1$. So we need to prove that

$$\forall n \geq 7.\ 0 \leq 5n + 10 \leq n^2$$

(2 points for a good argument for why this is true.)

We proceed by induction on n. As a base case, for $n = 7$ we have

$$0 \leq 45 \leq 49$$

Suppose $0 \leq 5n + 10 \leq n^2$ (the induction hypothesis). We need to show that it is also true for $n + 1$. That is, we need to show

$$0 \leq 5(n + 1) + 10 \leq (n + 1)^2$$

Simplifying this, we need to show

$$0 \leq 5n + 15 \leq n^2 + 2n + 1$$

Using the induction hypothesis, we can reduce this to

$$0 \leq 10 \leq 2n + 1$$

We know $n > 7$, so the right-hand side is larger than 15 and the proof is complete.

Name: _____

3. [10 points] What is the output of the following Python program?

```
A = [1,2]
B = A
A[0] = 3
B[1] = 4
print(B)
C = []
C.append((3,4))
print(C)
D = {(0,0): 'green', (0,1): 'blue', (1,0): 'red'}
print(D[(0,1)])
print((1,1) in D)
D[(1,0)] = 'purple'
print(D[(1,0)])
```

Solution: 2 points per line of correct output

```
[3, 4]
[(3, 4)]
blue
False
purple
```