Data Structures
CSCI C343, Fall 2015

Quiz 1

Name: ______________________

This quiz has 3 questions, for a total of 30 points.

1. $\boxed{\text{10 points}}$ What is the output of the following Python program?

```
A = [1,2]
A.append((3,4))
print(A)
print(1 in A)
print(4 in A)
B = A
C = A + B
A[1] = A[0]
print(B)
print(C)
```

> **Solution:** 2 points per line of correct output
>
> ```
> [1, 2, (3, 4)]
> True
> False
> [1, 1, (3, 4)]
> [1, 2, (3, 4), 1, 2, (3, 4)]
> ```

2. $\boxed{\text{9 points}}$ Suppose that `L` is a Python `list` (array) of length n. Categorize the worst-case execution time of the below expressions as either

   1. $O(1)$
   2. $O(\lg n)$
   3. $O(n)$
   4. $O(n^2)$

   Label each operation with the above item number.

   - `1000 in L`
   - `L.remove(0)`
   - `len(L)`

> **Solution:**
>
> - (3), `1000 in L` is $O(n)$, (3 points)
> - (3), `L.remove(0)` is $O(n)$, (3 points)
> - (1), `len(L)` is $O(1)$, (3 points)

3. $\boxed{\text{11 points}}$ Let $f(n) = n^2 + n + 10$ and $g(n) = n^2$. Give the definition of Big-O and prove that $f(n) \in O(g(n))$.

Solution: Definition of $O(g(n))$: (3 points)

$$O(g(n)) = \{f(n) \mid \exists n_0.\forall n \geq n_0.\exists c.\ 0 \leq f(n) \leq c\,g(n)\}$$

We need to choose a c such that cn^2 becomes greater than n^2+n+10 at some point. Ignore the $+10$ for the moment. For $n \geq 1$, we have $n^2 \geq n$, so

$$n^2 + n \leq n^2 + n^2 = 2n^2$$

So we choose $c = 2$ (3 points, there other valid choices). Next we need to find out at what point $2n^2$ is equal to or bigger than $n^2 + n + 10$, so we chart those out:

n	n^2+n+10	$2n^2$
0	10	0
1	12	2
2	16	8
3	22	18
4	30	32

So it looks like $n_0 = 4$ is a good choice (3 points, there other valid choices). We are now ready to give the proof.

To show that $n^2 + n + 10 \in O(n^2)$, we need to show that

$$\exists n_0.\forall n \geq n_0.\exists c.\ 0 \leq n^2 + n + 10 \leq c\,n^2$$

We choose $n_0 = 4$ and $c = 2$. So we need to prove that

$$\forall n \geq 4.\ 0 \leq n^2 + n + 10 \leq 2n^2$$

(2 points for a good argument for why this is true.)

We proceed by induction on n. As a base case, for $n = 4$ we have

$$0 \leq 30 \leq 32$$

Suppose $0 \leq n^2 + n + 10 \leq 2n^2$ (the induction hypothesis). We need to show that it is also true for $n+1$. That is, we need to show

$$0 \leq (n+1)^2 + (n+1) + 10 \leq 2(n+1)^2$$

Simplifying this, we need to show

$$0 \leq n^2 + 3n + 12 \leq 2n^2 + 4n + 2$$

Subtracting from both sides yields

$$0 \leq 10 \leq n^2 + n$$

which is true for $n > 4$.